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Data Analysis for Unsteady Turbulence Measurements over Airfoils

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Studies of unsteady flows about pitching airfoils and flows with unsteady freestream components require a consistent method of nonstationary data analysis for proper characterization of unsteady turbulent behavior. A method has been developed for data analysis of flows composed of high-frequency turbulence fluctuations superimposed upon a time-dependent low-frequency component or ensemble-averaged mean velocity with the goal of establishing a consistent method for the presentation of results from varying tests and flow situations. A nonstationary mean was determined by ensemble-averaging and frequency-domain low-pass filtering. Turbulence intensities were calculated based on a local mean using frequency-domain low-pass filtering methods. A nonstationary power spectral density estimator was developed to estimate the time-varying spectral characteristics of a turbulent boundary layer subjected to an unsteady freestream disturbance. A discussion concerning the choice of cutoff frequencies and digital filters is included.

Introduction

BECAUSE of the complex nature of turbulent flow, most experimental work in this field in the past has been restricted to steady (statistically stationary) turbulence. However, recent studies have begun to consider flows with time-varying turbulence properties as well as a low-frequency component or ensemble-average mean. De Grande et al.[1] describe the data processing of turbulence measurements around oscillating airfoils, and De Ruyck and Hirsch[2,3] continued the study of stalled boundary layers on a pitching airfoil. Uncertainties remain in determining or defining nonstationary "mean" velocities in these and in similar experiments, when a low-frequency component must be separated from a time-varying broad bandwidth high-frequency component.

Brendel[4] considered the effect of a pulsating freestream on the boundary-layer behavior of a low Reynolds number airfoil and presented time-mean and phase-locked velocity profiles. Crouch and Saric[5] used a "flying" or oscillating hot wire to study separating flows on a low Reynolds number airfoil. Howard[6] studied the periodic flow in a wing boundary layer immersed in a propeller slipstream. In studies such as these, the averaging algorithm for mean (or ensemble-average) velocity values and the determination of turbulence intensities or Reynolds stresses appear to be highly dependent on the particular measurement situation.

It is desirable to apply a nonstationary statistical data analysis method to the study of unsteady turbulent flows, with the purpose of establishing a consistent method for the presentation of results from different tests and flow situations. The current application is to attached boundary-layer flows, and desired are ensemble-average velocities, turbulence intensities, and spectral characteristics.

Background

Random processes may be classified as either stationary or nonstationary.[7] Nonstationary data may be classified as nonstationary mean, nonstationary variance, nonstationary frequency, and other specialized classes. A random process is nonstationary (the most general case) when the ensemble-averaged mean and autocorrelation function (and other ensemble properties) vary with time. The special classes of nonstationary data—nonstationary mean, variance, and frequency—imply that the mean, variance, or frequency content of the ensemble vary with time, but the other ensemble-averaged properties are time invariant.

The velocity (or other parameters of interest) of a turbulent fluid in motion can be modeled by defining the instantaneous velocity as a sum of a nonstationary mean velocity $\bar{u}(t)$ and a randomly fluctuating velocity $u'(t)$, with a zero mean

$$u(t) = \bar{u}(t) + u'(t) \tag{1}$$

where $u(t)$ is the velocity in the streamwise direction. The y and z velocity components are similarly defined. Steady turbulence is defined as a fluid flow with a constant mean velocity and a constant-variance high-frequency fluctuating velocity component [statistically stationary in $u'(t)$]. Unsteady turbulence for the special class of "nonstationary mean" is characterized as a fluid flow with a low-frequency time-dependent mean velocity and a constant-variance fluctuating velocity component. The challenge is to extract the nonstationary mean velocity $\bar{u}(t)$ from the measured instantaneous velocity $u(t)$.

Ensemble-averaging techniques may be used to determine the properties of nonstationary data when many repeated periodic time history records are available. For a single time history record, special techniques must be developed that apply to the limited classes of nonstationary data mentioned earlier. The current application relates to experiments where a limited number of data records are available (i.e., a statistically insufficient number of sample records for pure ensemble-averaging techniques to be effective). Such is the case for a majority of unsteady experiments that depend on the use of microcomputers for data acquisition and reduction. The necessary high sample rates for frequency resolution of the unsteady data limit the number of repeated records that can be recorded. The requirement of 30 h for a single boundary-layer traverse in the work of De Ruyck and Hirsch[3] underscores the problems in time-dependent boundary-layer studies. Other limitations, such as the time to hold a flight condition in boundary-layer studies on full-scale aircraft,[6] place a practical limit on the number of records that can be collected.

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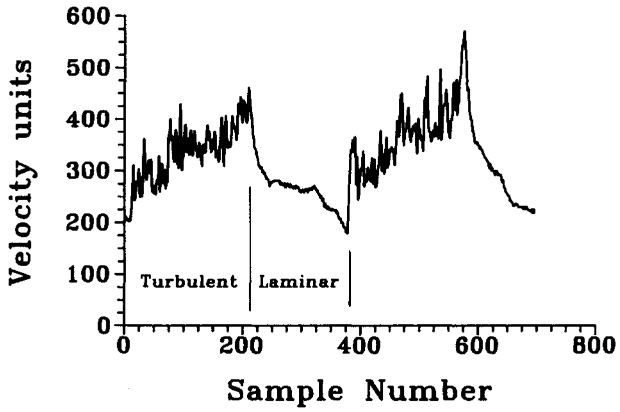

Fig. 1 One pulse from test case of 41 pulses; sample rate = 15 kHz.

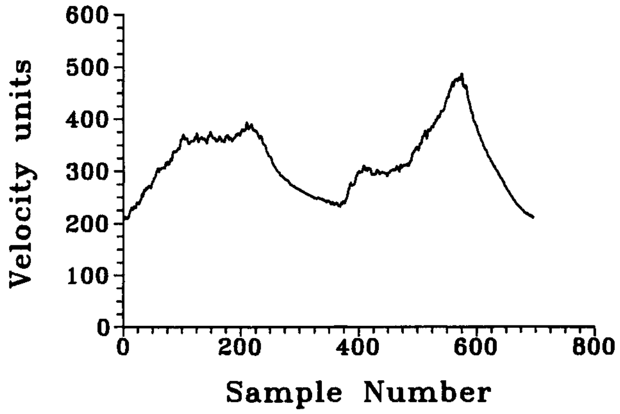

Fig. 2 Ensemble-average of the 41 propeller pulses contained in the test case.

Test Case

The sample record shown in Fig. 1 represents one velocity time history record from an ensemble of 41 records (propeller revolutions) taken in the boundary layer of an airfoil behind a spinning propeller. The data were digitized at 15 kHz, and approximately the first 700 samples are shown (for one propeller revolution) out of the total of almost 30,000 samples. The turbulent pulses are propeller blade wake passages, in between which the flow remains more or less laminar. A proximity transducer triggered by each revolution was used to mark the start of each cycle for digital processing.

Ensemble-Average Velocity

The initial problem is to estimate the unknown time-varying mean velocity from the measured instantaneous values. The classical approach is to ensemble average at fixed time t_i over N pulses (propeller blade wake passages):

$$\bar{u}(t_i) = \frac{1}{N}\sum_{k=1}^{N} u_k(t_i) \tag{2}$$

Samples are averaged across the ensemble (index k) at the same time point (index i) in the time-history record. The result of simple ensemble-averaging is shown in Fig. 2. The high-frequency component has been greatly reduced but not eliminated from the low-frequency waveform. This is a qualitative indication that insufficient samples are available for the ensemble-averaging operation, yielding a poor estimate of the mean velocity.

Ensemble-Averaged Velocity with Frequency-Domain Smoothing

An improved technique involves applying a computationally efficient frequency-domain low-pass digital filter to the ensemble-averaged mean velocity shown in Fig. 2. Using the convolution theorem to accomplish the low-pass filtering, the steps are as follows:

1) Transform the time series into the frequency domain using the discrete Fourier transform (DFT).

2) Multiply the transformed data (now a frequency series) by the frequency response function (magnitude characteristic) of the desired low-pass filter to remove the high-frequency content.

3) Then transform the data back into the time domain using an inverse DFT.

This technique has the advantage of computational speed over equivalent time-domain (moving-average) methods of low-pass filtering. Note that both ensemble-average and digital filtering based on the convolution theorem are linear operations. Thus, the result is independent of the order of these two operations. Performing ensemble-averaging before low-pass filtering, however, is more computationally efficient.

Low-Pass Filter Selection

The ideal frequency-domain low-pass filter transfer function is a step function with the step occurring at the desired cutoff frequency. The Fourier series representation of such a unit step transfer function is given by the equation[8]

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)e^{-j\omega} \tag{3a}$$

where the Fourier coefficients are given by

$$h(n) = \frac{1}{\pi n}\sin n(\omega_c) \tag{3b}$$

where ω is the radian frequency and ω_c the radian cutoff frequency. But, in a practical application, the nonrecursive digital filter transfer function represented by this infinite series must be truncated to a finite number of terms. This finite-length Fourier series representation of the ideal low-pass filter always results in overshoots and ripples in the transfer function, as illustrated in Fig. 3a. The undesirable response of ripples and overshoots in the frequency response function is known as the Gibbs' phenomenon.[9]

To improve the truncated series approximation of an ideal low-pass filter, the Gibbs' effect may be minimized through the application of a tapering (convolving) window function[8,9] to the transfer function in the frequency domain. A window function $w(n)$ is typically a monotonically decreasing function and attenuates the Fourier coefficients for the higher frequency terms:

$$\hat{h}(n) = h(n)w(n) \tag{4}$$

This application reduces the amplitudes of the ripples and overshoots and smooths out the truncated transfer-function series at the cost of widening the transition band of the filter (Fig. 3b). The benefit of working in the frequency domain is that multiplication in the frequency domain is equivalent to convolution in the time domain.

The net result is an improved approximation to the ideal filter. Through the application of a variety of smoothing windows given by Harris,[10] and with the variation of the number of Fourier coefficients retained, a multitude of low-pass filter transfer functions may be designed that give good approximations to the ideal filter with a reasonable number of Fourier coefficients. Additionally, more advanced recursive digital filter techniques could be applied if desired.

In this application, a very simple parabolic transfer function was chosen for the low-pass filter and no smoothing window

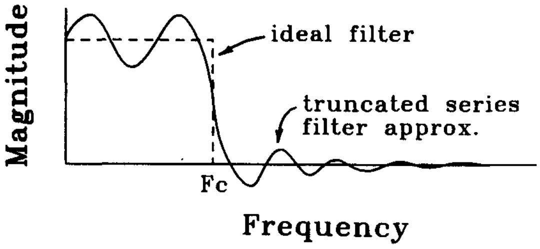

Fig. 3a Truncated Fourier series representation of an ideal filter, showing Gibbs' effect.

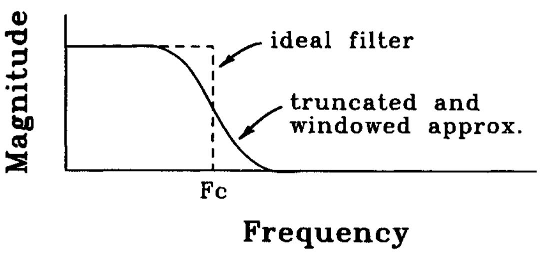

Fig. 3b Low-pass filter transfer function after application of a smoothing window.

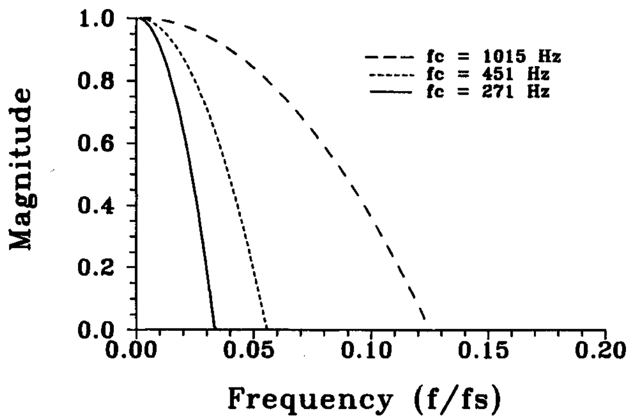

Fig. 4 Parabolic transfer function of the frequency-domain smoothing filter for three different cutoff frequencies.

was used. The transfer function of the low-pass filter is defined by

$$H(f/f_s) = \max\left[1 - \left(\frac{L}{M} - J\right)^2, 0\right]$$

$$J = 0, 1, 2, \ldots, M \tag{5}$$

where L is the integer parabola width at zero magnitude that determines the filter cutoff frequency f_c (defined at -3dB), M is the (integer power of 2) number of data points in the frequency series, J is the frequency bin index running from 0 to M, and $H(f/f_s)$ is the magnitude of the frequency response parameterized by the sample rate f_s. Figure 4 shows the frequency response characteristics of this low-pass filter.

Results from applying 8-point (f_c = 1015 Hz), 25-point (f_c = 325 Hz), and 51-point (f_c = 160 Hz) low-pass filters are shown in Figs. 5a–5c. By combining digital filtering with ensemble averaging, the previously described algorithm provides an improved analysis technique for estimating the nonstationary mean velocity when the number of sample records in the ensemble is statistically insufficient to accomplish pure ensemble averaging. The cutoff frequency of the digital filter must be carefully selected, as noted later. For the current example, propeller pulses were generated at a frequency of 40 Hz; with the a priori knowledge that spectral peaks in transitional boundary layers lie at approximately 800 Hz and

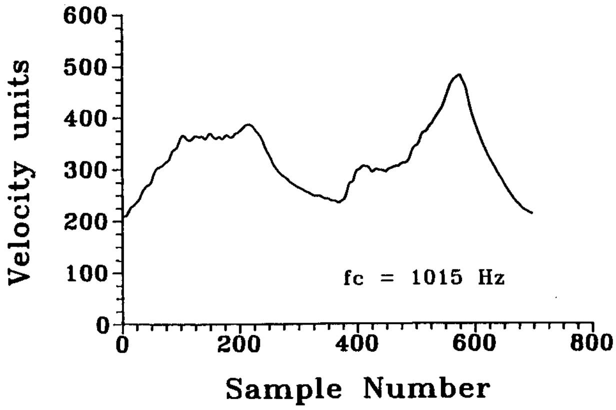

Fig. 5a Result of test case ensemble-averaging and smoothing with a low-pass digital filter; cutoff frequency, 1015 Hz; sample rate, 15 kHz.

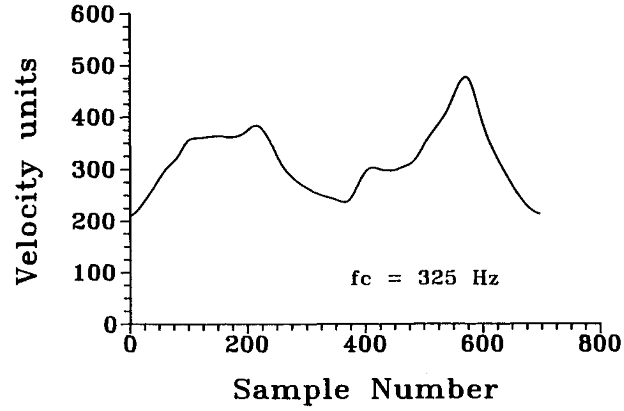

Fig. 5b Result of test case ensemble-averaging and smoothing with a low-pass digital filter; cutoff frequency, 325 Hz; sample rate, 15 kHz.

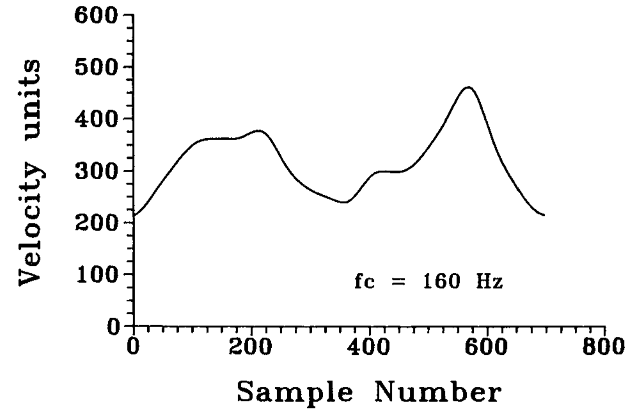

Fig. 5c Result of test case ensemble-averaging and smoothing with a low-pass digital filter; cutoff frequency, 160 Hz; sample rate, 15 kHz.

that most freestream energy is contained below 200 Hz,[11] a cutoff frequency between these values is desirable.

Turbulence Intensity

Turbulence intensity Tu is usually defined as the root mean square of the fluctuating velocity component u' normalized by a reference constant velocity U_e. For steady turbulence, this is very similar to the statistical variance. But for nonstationary turbulence, the fluctuating velocity component may be defined as the difference between the instantaneous velocity $u(t)$ and the local smoothed velocity $\bar{u}(t)_{\text{local}}$ rather than the ensemble-averaged mean velocity. This following definition of turbulence prevents the pulse-to-pulse variations in the time-dependent local mean from incorrectly being included in the turbulence intensity determination.

$$Tu = \frac{\sqrt{\frac{1}{N}\sum_{k=1}^{N}[u_k(t_i) - \bar{u}_k(t_i)_{\text{local}}]^2}}{U_e} \qquad (6)$$

Turbulence Intensity Algorithm

The algorithm presented here differs from the classical definition only in the use of low-pass frequency-domain filtering to obtain the smoothed (local mean) velocity. Cases were run at the same cutoff frequencies as used for the low-pass-filtered

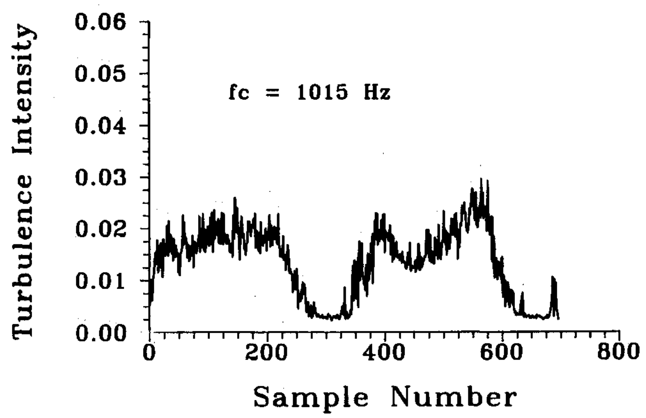

Fig. 6a Result of test case turbulence intensity using digital low-pass filtering to compute local mean; cutoff frequency, 1015 Hz; sample rate, 15 kHz.

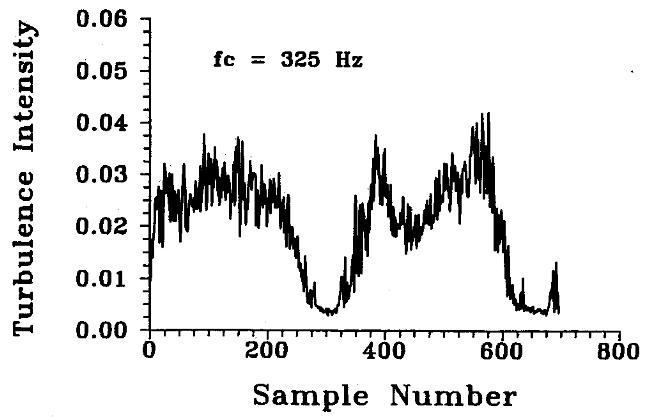

Fig. 6b Result of test case turbulence intensity using digital low-pass filtering to compute local mean; cutoff frequency, 325 Hz; sample rate, 15 kHz.

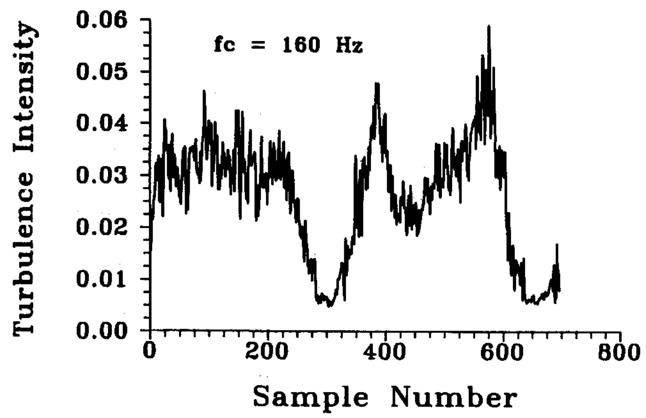

Fig. 6c Result of test case turbulence intensity using digital low-pass filtering to compute local mean; cutoff frequency, 160 Hz; sample rate, 15 kHz.

ensemble-average velocity examples—at 1015, 325 and 160 Hz—and are shown in Figs. 6a–6c. These data show the time-dependent, periodic nature of the unsteady turbulent boundary layer immersed in a propeller slipstream. The values of turbulence intensity can be seen to be much more sensitive to the chosen value of cutoff frequency than were the ensemble-averaged mean velocities, as would be expected. A decrease in cutoff frequency from 325 to 160 Hz resulted in a change in perceived turbulence intensity levels of approximately 15%. The cutoff frequency must be carefully selected by the aerodynamicist to be consistent with the chosen low-frequency-mean empirical model and the physical nature of the unsteady turbulent boundary layer being considered.

An alternate method could be to use Parseval's theorem,[12] which states that the energy contained in a time-domain signal is conserved when the signal is transformed into the frequency domain, as a basis for a computationally efficient, frequency-domain-based algorithm to calculate turbulence intensity. In this manner, a definite bandwidth of the turbulent energy can be specified. Bradshaw[13] discusses requirements for the filter bandwidth in spectral analysis.

Spectral Analysis

Of interest in any type of unsteady turbulence environment is the time-varying frequency spectra of the nonstationary data. The spectral estimation scheme used is based on the Welch method described by Marple.[14] This method provides an efficient FFT-based algorithm and does not depend on the assumption of a specific parametric model for the input data. The basic Welch spectral estimation algorithm is based on the second-order statistics of stationary data. Although the unsteady turbulence data presented here are not stationary, it is assumed that local stationarity is maintained over a short time segment within each propeller pulse. Therefore, some background is first provided on the Welch spectral estimation algorithm for stationary data.

Spectral Estimation of Stationary Data

The spectral estimation algorithm developed by Welch is based on the classical "segment and average" method. This method produces a (nearly) minimum variance spectral estimate. The variance of the Welch PSD (power spectral density) estimator is roughly inversely proportional to the number of overlapping segments. A large number of segments is therefore desirable, except that a large number of segments reduces the frequency resolution (the discrete frequency bins) for a given fixed quantity of data. The tradeoff is to choose an adequate frequency resolution with the largest number of segments possible to minimize variance in the spectral estimate. At least 14 overlapping segments (with 50% overlap) is desirable for a reasonably low variance spectral estimate.

The approach is to divide the time-history record into a number of shorter overlapping segments, apply a tapering window to each segment, and compute the "raw" power spectral density of each segment. The PSD estimate for each segment is then corrected for the tapering window bias and all of the segments are averaged to produce the final Welch PSD estimate. The purpose of the tapering window function is to minimize the frequency "leakage" effect of the Gibbs' phenomenon caused by computing a DFT of the truncated time series.[8,9] In general, the truncated (finite) time series fails to contain an exact integer number of cycles in the sampled time slice at a given frequency. Thus, individual frequency components of the time slice are discontinuous in periodic extension and result in a broadband spectrum. The previous discussion of low-pass filters considered the use of windows applied to the filter transfer function in the frequency domain to minimize the Gibbs' effect ripples and to achieve a better approximation to an ideal filter. Here, a tapering window is applied directly to the truncated time series to minimize Gibbs' effect frequency leakage in the frequency spectrum. Multiplication of a time series by a tapering (truncating) window in the time domain is equivalent to convolution in the frequency domain.

A window function is selected based on a tradeoff between frequency resolution loss (main lobe width) and the amount of leakage suppression (highest side lobe level).[14] The Hamming window or the three-term Blackman-Harris window[10] provides a good compromise; the Hamming spectral window was used in the current work. An example for stationary data is given by Johnson[15]; only an example for nonstationary data will be presented here.

Spectral Estimation of Nonstationary Data

With nonstationary data, the frequency content of the data varies with time. The data being considered are the ensemble of 41 records (pulses) discussed previously. The approach for nonstationary spectral estimation is to define identical short segments across the ensemble of pulses. A short time segment from the first record is windowed using an appropriate tapering window, and a raw PSD estimate is computed for the segment. This estimate is taken to represent the spectrum at the center time point of the segment. A raw PSD estimate is computed for the corresponding short time segment in every other record across the ensemble, and the collection is averaged and corrected for the tapering window to produce the final "instantaneous" spectral estimate for the ensemble at the time represented by the center of the segment. Analogously to averaging raw PSDs from overlapping segments in the previous stationary case, the raw segment PSDs are ensemble averaged to obtain a low-variance spectral estimate for a short time segment of the ensemble. This process is repeated for other segments, leading to the construction of time-varying spectral characteristics of the complete propeller pulse cycle.

An assumption implicit in this algorithm is that the data are stationary over the short time segment. This assumption appears reasonable, due to the large separation of frequencies of the wake passage and the turbulence. However, the time segment must be long enough to achieve the desired resolution in the frequency spectrum. The short time segment should not cross a transition point between laminar and turbulent flow, thus violating the assumption of stationarity over the short segment. If a segment cannot be located without including a transition point, then the transition point should be located near the beginning or end of the segment such that the tapering window will minimize the effect of the nonstationary abrupt transition.

For the test case presented here, a 256-point segment was chosen as a compromise between short segment length and desired frequency resolution, providing 128 positive frequencies in the frequency spectrum. The segment length corresponds to approximately one-third of the length of an individual pulse record, and it was in three parts that the time-dependent flow was considered: an undisturbed regime, a highly turbulent regime, and a recovery regime. An application of the described algorithm was carried out by Renoud and Howard,[16] who defined these three flow regimes for the case of the boundary-layer response to the flow downstream of a spinning rod. The decision to divide the pulse into three regimes was also related to the sampling frequency. Options included increasing the sample rate, decreasing the segment width to 128 points, or maintaining the 256-point segment, but with overlapping segments. An increased sample rate would allow a 256-point segment to represent a smaller time interval and thus increase the resolution in time of the nonstationary spectrum. The ensemble-average velocity of each individual segment was removed to allow consideration of the turbulence spectral characteristics. Also, the Hamming tapering window was selected from the options available[10] to compute the frequency spectra shown here.

The results of applying the spectral estimation algorithm to the test case are shown in Fig. 7. The first ensemble segment is centered on a time point that characterizes the turbulent

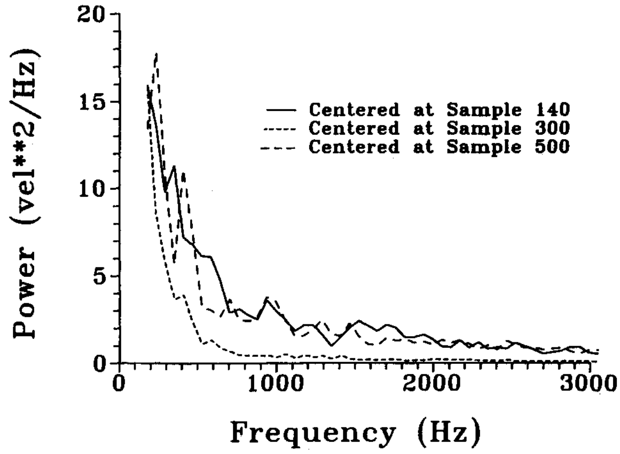

Fig. 7 Spectral estimate of three ensemble segments from test case; Hamming window, segment mean removed, sample rate 15 kHz.

flow. The second segment represents undisturbed laminar flow, and the third a return to turbulent flow. The power level for the laminar segment is well below the other two, illustrating the time-dependent nature of the frequency spectra of nonstationary data.

All of the algorithms described in this paper have been implemented in FORTRAN source code and are available from the second author by request.

Summary

A data analysis method was developed to allow for a consistent description of flows composed of high-frequency turbulence fluctuations superimposed on a low-frequency component or ensemble-average mean velocity. A method was outlined to characterize a boundary layer subjected to time-dependent turbulent freestream disturbances in terms of ensemble-average mean velocities, turbulence intensities, and time-dependent spectral characteristics.

A nonstationary mean was determined by ensemble-averaging and then frequency-domain low-pass filtering. The computation time savings over the reverse operation was 75% on a personal computer. Various low-pass filters can be implemented. Cutoff frequencies can be chosen according to the low- and high-frequency bandwidths in the experiment.

Turbulence intensities were calculated based on a local mean using frequency-domain low-pass filtering methods. Various digital filters with differing frequency response characteristics can be implemented as desired.

A nonstationary (ensemble-averaging) spectral estimator was developed based on the classical Welch periodogram algorithm. The nonstationary spectral estimator was used to estimate the time-varying spectral characteristics of the turbulent boundary layer velocity data for a representative test case.

Recommendations

The procedure discussed in this study is not intended to provide a definitive approach to unsteady turbulence data analysis but to promote the continued transfer of advanced signal processing techniques to the study of complex aerodynamic flows. Recommendations for further work in this field include implementation of alternate frequency-domain low-pass filters in the smoothing algorithm to better approximate the ideal low-pass filter. Also, an alternate algorithm based on Parseval's theorem to calculate turbulence intensity in the frequency domain could be developed and implemented. Additionally, depending on the flow situation under study, other empirical models of nonstationary turbulent fluid flow could be examined using the special classes of nonstationary data suggested by Ref. 7, such as a nonstationarity in variance, a nonstationarity in frequency, or a three-part decomposition

$$u(t) = \bar{u}(t) + \tilde{u}(t) + u'(t) \tag{8}$$

where $\bar{u}(t)$ is a deterministic mean, and $\tilde{u}(t)$ is a deterministic periodic velocity component.

It is hoped that improved nonstationary statistical signal processing methods can be applied with the goal of establishing consistent and mathematically rigorous methods for analyzing results from unsteady turbulent flows.

For spectral analysis of related flows, the use of analog bandpass filtering before digital sampling is recommended to eliminate the strong low-frequency propeller-pulse spectrum lines and to eliminate high-frequency noise above the range of interest, thus removing potential errors due to data aliasing.

Acknowledgments

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References

[1]De Grande, G., Haverbeke, A., and Hirsch, Ch., "Digital Processing of Unsteady Periodic Signals with Application to the Turbulence Structure Around Oscillating Airfoils," Institute of Electrical and Electronics Engineers, IEEE Publication 79 CH 1500-8 AES, Piscataway, NJ, Sept. 1979, pp. 273–283.

[2]De Ruyck, J., and Hirsch, C., "Turbulence Structure in the Wake of an Oscillating Airfoil," Vrije Universiteit, Dept. of Fluid Mechanics, Report VUB-STR-12, Brussels, Belgium, Nov. 1981.

[3]De Ruyck, J., and Hirsch, C., "Instantaneous Flow Field Measurements of Stalled Regions on an Oscillating Airfoil," AIAA Paper 84-1565, June 1984.

[4]Brendel, M., "Experimental Study of the Boundary Layer on a Low Reynolds Number Airfoil in Steady and Unsteady Flow," Ph.D. Dissertation, Univ. of Notre Dame, Notre Dame, IN, May 1986.

[5]Crouch, J. D., and Saric, W. S., "Oscillating Hot-Wire Measurements Above an FX63-137 Airfoil," AIAA Paper 86-0012, Jan. 1986.

[6]Howard, R. M., "An Investigation of the Effects of the Propeller Slipstream on a Wing Boundary Layer," Ph.D. Dissertation, Texas A & M Univ., College Station, TX, May 1987.

[7]Bendat, J. S., and Piersol, A. G., *Random Data Analysis and Measurement Procedures*, 2nd ed., Wiley, New York, 1986, p. 427.

[8]Strum, R. D., and Kirk, D. E., *First Principle of Discrete Systems and Digital Signal Processing*, Addison Wesley, Reading, MA, 1988, pp. 536–546.

[9]Hamming, R. W., *Digital Filters*, 2nd ed., Prentice-Hall, Englewood Cliffs, NJ, 1983, p. 93.

[10]Harris, F. J., "On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform," *Proceedings of the IEEE*, Vol. 66, No. 1, 1978, pp. 51–83.

[11]Landrum, D. B., and Macha, J. M., "Influence of a Heated Leading Edge on Boundary Layer Growth, Stability, and Transition," AIAA Paper 87-1259, June 1987.

[12]Brigham, E. O., *The Fast Fourier Transform*, Prentice-Hall, Englewood Cliffs, NJ, 1974, p. 130.

[13]Bradshaw, P., *An Introduction to Turbulence and Its Measurement*, Pergamon, New York, 1971, pp. 148–152.

[14]Marple, S. L., *Digital Spectral Analysis with Applications*, Prentice-Hall, New York, 1987.

[15]Johnson, D. K., "A Data Analysis System for Unsteady Turbulence Measurements," Masters Thesis, Naval Postgraduate School, Monterey, CA, Sept. 1988.

[16]Renoud, R. W., and Howard, R. M., "Airfoil Boundary Layer Response to an Unsteady Turbulent Flowfield," *AIAA Journal*, Vol. 28, No. 11, 1990, pp. 1894–1900.